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Strain analysis using the shape of expected and observed continuous frequency distributions

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Abstract—An initial attempt to develop a method of strain analysis based on shape modifications to continuous frequency distributions of structural data during deformation is described. The method uses the dimensionless coefficients of skewness (β_1 = skewness^2/variance^3) and kurtosis (β_2 = kurtosis/variance^2) and although the justification for adopting these parameters is complex, the actual calculation of β_1 and β_2 is relatively simple. Different frequency distributions can be accurately distinguished by plotting graphs of β_1 against β_2. Since the effect of strain on a frequency distribution is to modify its shape, theoretically determined shape modifications can be followed on β_1 vs β_2 graphs for increasing strain and hence the graphs are automatically contoured in terms of strain. The strains involved in natural deformations can then be estimated by plotting on the graphs the β_1, β_2 values for observed continuous frequency distributions. Various examples of the application of this technique are discussed using data from the literature.

INTRODUCTION

THIS brief contribution outlines an initial attempt to develop a method of strain analysis using the shape of continuous frequency distributions. Distribution shape is preferred to other more commonly employed statistics (e.g. mean, variance, median and mode) since it is usually characteristic for a particular distribution and can be represented simply by a pair of dimensionless coefficients. The philosophy behind this approach is that any initial continuous frequency distribution of structural elements (e.g. orientations, spacings, lengths, etc.) is transformed by progressive deformation into a *strain-modified* distribution, the shape of which is a function of the type and magnitude of the deformation. In most cases distribution shape can be described in terms of the dimensionless coefficients of skewness and kurtosis (Elderton & Johnson 1969, Harr 1977),

$$\begin{aligned} \beta_1 &= \text{skewness}^2/\text{variance}^3 \\ \beta_2 &= \text{kurtosis}/\text{variance}^2. \end{aligned} \qquad (1)$$

The former coefficient defines the symmetry of the distribution and the latter its peakedness. The justification for using these two parameters is presented in the next section.

THEORETICAL JUSTIFICATION: DISTRIBUTION CLASSIFICATION

The following theoretical discussion is based on the classification of continuous frequency distributions suggested by Karl Pearson (for a complete review of his work see Elderton & Johnson 1969).

In general, continuous frequency distributions can be considered to begin at zero, rise to a maximum and then fall away (often at a different rate) such that there is usually high contact at the ends of the distribution. To represent this behaviour mathematically a series of equations $y = \mathrm{f}(x)$, $y = \phi(x)$ is required in which $\mathrm{d}y/\mathrm{d}x = 0$ for specific values of x (e.g. at the maximum and at the end of the curve where there is contact with the x axis). This suggests that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y(x + x_0)/\mathrm{F}(x), \qquad (2)$$

since $\mathrm{d}y/\mathrm{d}x = 0$ if $y = 0$ or $x = x_0$; the former corresponds to the contact with the x axis at one end of the curve and the latter represents the curve maximum if x_0 is defined as the distance between the origin and the mode. Rearranging equation (2) and expanding $\mathrm{F}(x)$ by Maclaurin's theorem yields

$$(b_0 + b_1x + b_2x^2 + \ldots)\frac{\mathrm{d}y}{\mathrm{d}x} = y(x + x_0), \qquad (3)$$

in which b_0, b_1, b_2, etc. are constants.

The majority of continuous frequency distributions can be generated from equation (3) firstly by multiplying by x^n and then integrating with respect to x,

$$x^n(b_0 + b_1x + b_2x^2 + \ldots)y - \int [nb_0x^{n-1} + (n+1)b_1x^n + (n+2)b_2x^{n+1} + \ldots]y\mathrm{d}x = \int yx^{n-1}\mathrm{d}x + \int yx_0x^n\mathrm{d}x. \qquad (4)$$

The expression $x^n(b_0 + b_1x + b_2x^2 + \ldots)y$ vanishes at the ends of a frequency curve. Thus by writing $\mu_n = \int yx^n\mathrm{d}x$ and rearranging, equation (4) becomes

$$x_0\mu_n + nb_0\mu_{n-1} + (n+1)b_1\mu_n + (n+2)b_2\mu_{n+1} + \ldots = -\mu_{n+1}. \qquad (5)$$

For successive positive integer values of n from zero to s, $s + 1$ equations are formed which allow the constants to be determined:

$$\begin{aligned} x_0\mu_0 + 0b_0 + 1b_1\mu_0 + 2b_2\mu_1 &= -\mu_1 \\ x_0\mu_1 + 1b_0\mu_0 + 2b_1\mu_1 + 3b_2\mu_2 &= -\mu_2 \\ x_0\mu_2 + 2b_0\mu_1 + 3b_1\mu_2 + 4b_2\mu_3 &= -\mu_3 \\ x_0\mu_3 + 3b_0\mu_2 + 4b_1\mu_3 + 5b_2\mu_4 &= -\mu_4. \end{aligned} \tag{6}$$

It is usually convenient to make the mean the origin of the distribution, in which case x_0 becomes the distance between the mean and the mode and it is necessary to amend the other terms accordingly. Assuming grouped data, these become

$$\begin{aligned} \mu_1 &= \frac{1}{N}\sum_{i=1}^{m}\{f_i x_i\} = \text{sample mean} \\ \mu_2 &= \frac{1}{N}\sum_{i=1}^{m}\{f_i(x_i - \mu_1)^2\} = \text{sample variance} \\ \mu_3 &= \frac{1}{N}\sum_{i=1}^{m}\{f_i(x_i - \mu_1)^3\} = \text{sample skewness} \\ \mu_4 &= \frac{1}{N}\sum_{i=1}^{m}\{f_i(x_i - \mu_1)^4\} = \text{sample kurtosis} \end{aligned} \tag{7}$$

in which m is the number of data groups, x_i and f_i are the midpoint and frequency of the ith group and N is the sample size. It is also convenient to treat μ_0 as unity and together these simplifications lead to a set of simultaneous equations which can be solved and substituted into equation (3) to give

$$\frac{1}{y}\frac{dy}{dx} = \frac{x + (M_1/M_2)}{(M_3 + M_1x + M_4x^2)/M_2}$$

$$\text{with: } \begin{aligned} M_1 &= \mu_2^{1/2}\beta_1^{1/2}(\beta_2 + 3) \\ M_2 &= 2(5\beta_2 - 6\beta_1 - 9) \\ M_3 &= \mu_2(4\beta_2 - 3\beta_1) \\ M_4 &= 2\beta_2 - 3\beta_1 - 6 \end{aligned} \tag{8}$$

where $\beta_1 = \mu_3^2/\mu_2^3$, $\beta_2 = \mu_4/\mu_2^2$ [in agreement with equation (1)] and $x_0 = \mu_2^{1/2}\beta_1^{1/2}(\beta_2 + 3)/2(5\beta_2 - 6\beta_1 - 9)$; note that $\mu_2^{1/2}$ is the standard deviation.

By inserting the values of the sample moments into equation (8) it is possible to obtain a formula representative of the distribution data. However, this formula is not of exactly the same form as the original data. A truly representative formula can be obtained if $(x + x_0)/(b_0 + b_1x + b_2x^2 \ldots)$ is integrated. This is possible by recognising equation (8) as a general expression for integration and observing that the form the integral takes depends on the particular values of the coefficients of x in the denominator for

$$b_1x + b_2x^2 = b_2\left[x - \frac{-b_1 + (b_1^2 - 4b_0b_2)^{1/2}}{2b_2}\right] \times \left[x - \frac{-b_1 - (b_1^2 - 4b_0b_2)^{1/2}}{2b_2}\right]. \tag{9}$$

The basis for fixing the particular form is the same as that for the nature of the roots of the equation $b_0 + b_1x + b_2x^2 = 0$; that is $b_1^2/(4b_0b_2)$. By substituting from equation (8) this gives $[\beta_1(\beta_2 + 3)^2]/[4(2\beta_2 - 3\beta_1 - 6)(4\beta_2 - 3\beta_1)]$ which is usually denoted by the symbol K.

Pearson called K the criterion and used it to classify the different types of continuous frequency distributions (although he found it useful to employ other 'criteria' as well; see Elderton & Johnson 1969). However, since all the 'criteria' are functions of the parameters β_1 and β_2 it is possible to distinguish between different distributions using these terms; Harr (1977) suggested they should be plotted against each other (Fig. 1). It is proposed that plots of β_1 vs β_2 are capable of distinguishing between initial and strain-modified distributions and hence can be used to give estimates of the magnitude of finite strains and also the types of strains involved. The next section discusses this approach.

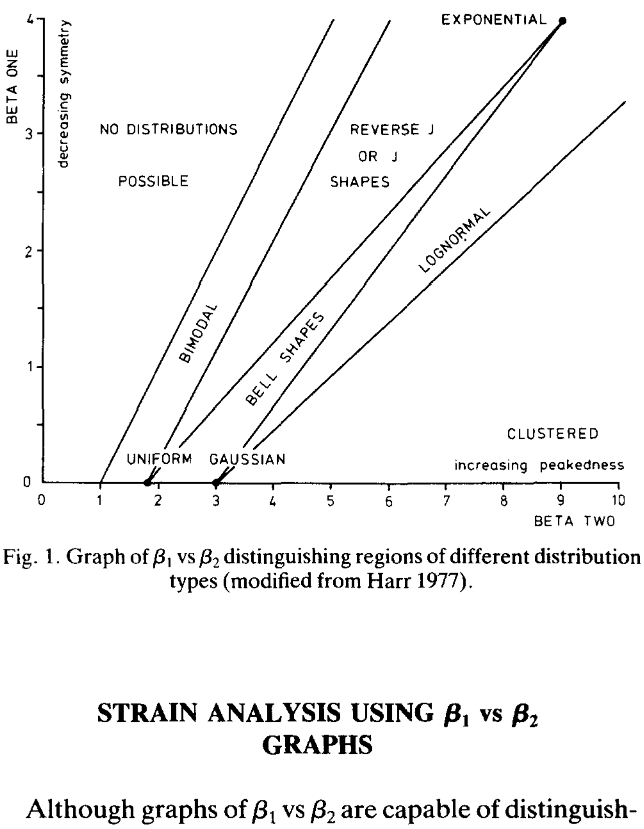

Fig. 1. Graph of β_1 vs β_2 distinguishing regions of different distribution types (modified from Harr 1977).

STRAIN ANALYSIS USING β_1 vs β_2 GRAPHS

Although graphs of β_1 vs β_2 are capable of distinguishing between different continuous frequency distributions, in this simple form they do not reveal any information on strain. The use of such graphs in strain analysis requires that the effects of different types and amounts of strain on initial frequency distributions are known. The values of β_1 and β_2 for the various theoretical strain-modified distributions would then be calculated and hence the graphs contoured in terms of strain magnitudes, with different graphs constructed for different types of strain. It would then be a relatively simple matter to calculate the β_1 and β_2 values for natural data and hence to determine the type and magnitude of the natural finite strains.

An important pre-requisite for this technique is that the theory exists for the modification of an initial distribution by a particular type of strain. To date, only two such theories are known in the literature and concern the effects of homogeneous irrotational strains on initially Gaussian and uniformly distributed data.

Strain-modified Gaussian distributions

Sanderson (1973) studied the effect of homogeneous irrotational strain on orientation data with an initial Gaussian distribution. He showed that the Gaussian distribution (maintaining the present notation)

$$F = \frac{N}{(2\pi\mu_2)^{1/2}} \exp\left(-\frac{1}{2\mu_2}(90 - \theta)^2\right) \quad (10a)$$

is modified to

$$F' = \frac{N}{(2\pi\mu_2)^{1/2}} \exp\left(-\frac{1}{2\mu_2}(90 - \theta)^2\right) \times \frac{[(X/Y)^2 \cos^2(\theta - \phi) + \sin^2(\theta - \phi)]^{3/2}}{(X/Y)} \quad (10b)$$

in which N is the sample size, $\mu_2^{1/2}$ is the standard deviation of the initial distribution, θ is the angle between the observed direction and the extension direction (X) of the finite strain ellipsoid ($X \geqslant Y \geqslant Z$), ϕ is the angle between the initial mean and the normal to the extension direction and X/Y is the strain ratio. For $\phi = 0$ the initial distribution is perpendicular to X but as the strain increases the distribution spreads and then splits into two maxima symmetrical about X (Sanderson 1973; see Fig. 2a). For $\phi \neq 0$ the initial distribution is oblique to X and as the strain increases the peak rotates towards X with a subsidiary peak developing after a certain value of strain (Sanderson 1973; see Fig. 2b).

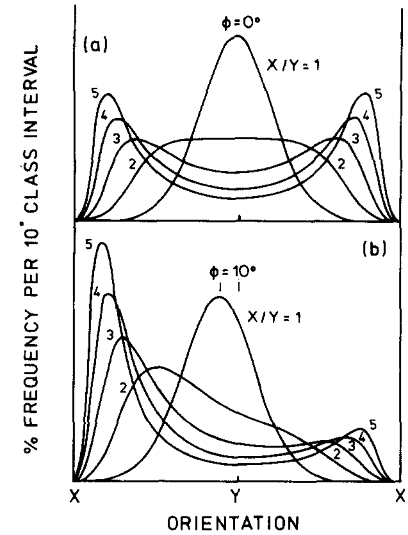

Fig. 2. Homogeneous irrotational strain-modified Gaussian distributions (modified from Sanderson 1973); X/Y is the strain-ratio, $\mu_2^{1/2}$ is the standard deviation of the initial distribution and ϕ is the angle between the mean direction of the initial distribution and the normal to the extension direction (X).

The β_1 and β_2 values of strain-modified Gaussian distributions have been determined and plotted on a graph of β_1 vs β_2 for different strains and values of ϕ but constant standard deviation (Fig. 3). For $\phi = 0$ all the distributions are symmetrical and therefore plot along the β_2 axis but since increasing strain causes a change from unimodality to bimodality there is a gradual migration towards smaller values of β_2. For $\phi \neq 0$ the distributions are not symmetrical and plot further away from the β_2 axis with increasing ϕ, lying on distinctive curves for each value of ϕ. Note that while ϕ remains small the strain-modified distributions eventually become bimodal, but for larger values of ϕ they remain unimodal.

Having drawn the distinctive curves for each value of ϕ it is possible to join together the points of equal strain (Fig. 3). For small strains these are slightly curved but for larger strains ($X/Y >\sim 3$) they may be considered as linear. This operation therefore contours the β_1 vs β_2 graphs in terms of strain and makes them extremely useful in the estimation of finite homogeneous irrotational strains as the following examples show.

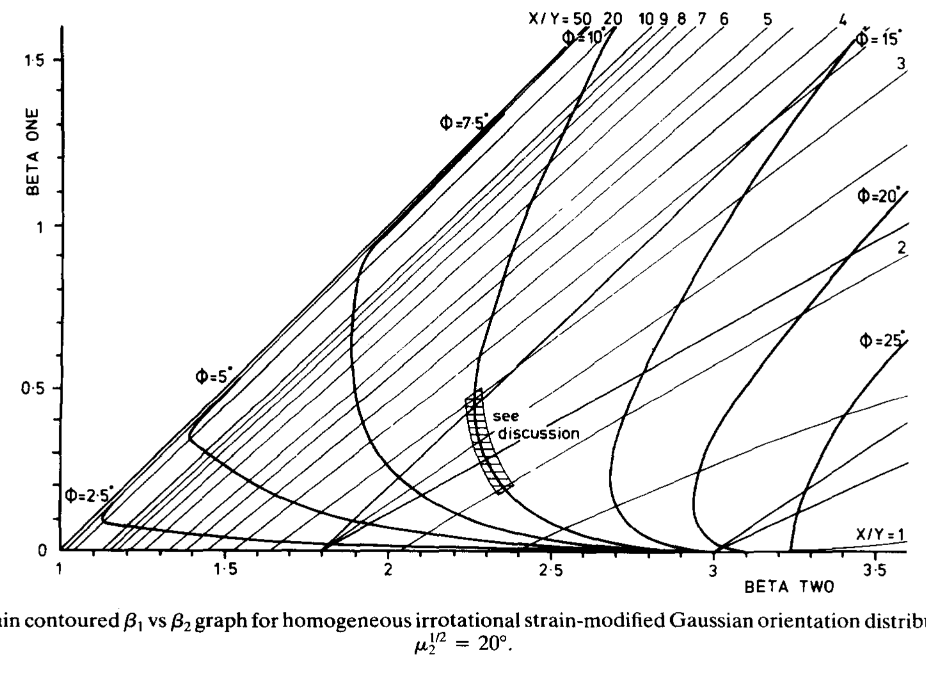

Fig. 3. Strain contoured β_1 vs β_2 graph for homogeneous irrotational strain-modified Gaussian orientation distributions with $\mu_2^{1/2} = 20°$.

Table 1. Comparison between strain magnitudes obtained using Sanderson's (1973) technique for homogeneous irrotational strain-modified Gaussian orientation distributions and those using β_1 vs β_2 graphs; $\mu_2^{1/2}$ = standard deviation, X/Y = strain ratio; ϕ = angle between initial mean and normal to final extension direction (i.e. Y-axis)

	Sanderson 1973	Roberts & Sanderson 1974			
	Observed fold axis distribution	Original fold axis distribution	Strain modified fold axis distribution		
	Boscastle (Fig. 4) $\mu_2^{1/2} = 20°$	Craignish (Fig. 5a) $\mu_2^{1/2} = 21°$	Knapdale steep belt (Fig. 5b) $\mu_2^{1/2} = 20°$	Loch Tay inversion (Fig. 5c) $\mu_2^{1/2} = 20°$	Aberfoyle Anticline (Fig. 5d) $\mu_2^{1/2} = 25°$
Sanderson					
X/Y	4–5	very low (~1)	3.5	4	4
ϕ	5°	11.6°	5°	15°	3°
This paper					
X/Y	6.5	1.2	4.5	5.75	6.75
ϕ	8°	26.5°	4.5°	17°	6°
β_1	0.765	0.112	0.171	3.922	0.298
β_2	2.012	3.416	1.589	5.748	1.484

Examples of strain-modified Gaussian distributions

Sanderson (1973) applied his model to fold axes which occur oblique to the regional trend. He assumed that the fold axes originally had a Gaussian distribution ($\mu_2^{1/2}$ = 20°) about the Y axis of the strain ellipsoid but were subsequently rotated towards the X direction by further (stretching) deformation within the axial planes of the folds. The initial distribution was consequently modified by passive rotation and relative elongation of fold axes within the XY plane of the deformation to produce a slightly asymmetric final distribution with a subsidiary maximum (Fig. 4), suggesting that $\phi > 0°$. Sanderson therefore used $\phi = 5°$ and thus determined that $4 < X/Y < 5$. The method described here, using a strain-contoured β_1 vs β_2 graph for $\mu_2^{1/2} = 20°$, gives $\phi = 8°$ and $X/Y = 6.5$ (Table 1).

In a subsequent use of Sanderson's model Roberts & Sanderson (1974) have analysed the variation in patterns of fold axis distributions across the Scottish SW Highlands, which they attribute to varying amounts of deformation and to the initial attitude of the mean fold axis relative to the finite strain axes. They argued that the folds formed with a mean fold axis nearly perpendicular to the stretching direction and that this original Gaussian distribution can still be recognised in areas where the subsequent deformation was low (e.g. at Craignish, see Fig. 5a). There is a progressive modification of the initial distribution southeastwards across the region from Craignish through the Knapdale steep belt and Loch Tay inversion to the Aberfoyle Anticline (Fig. 5). There is also some variation in the values of $\mu_2^{1/2}$ and ϕ, especially near Loch Tay where the initial mean fold axis direction was somewhat oblique to the stretching direction (Table 1). Roberts & Sanderson (1974) showed that there must have been a progressive increase in strain southeastwards from Craignish ($X/Y \approx 1$) towards the Loch Tay inversion and Aberfoyle Anticline ($X/Y \approx 4$). The technique described here has been applied to the data shown in Fig. 5, using strain-contoured β_1 vs β_2 graphs for the different values of $\mu_2^{1/2}$, and shows a similar trend, although the strain continues to increase into the Aberfoyle Anticline (Table 1). The present technique also recognises the more oblique initial fold-axis distribution in the Loch Tay inversion and suggests that the distribution at Craignish is not the initial Gaussian distribution but is the result of very slight stretching ($X/Y = 1.2$) of an oblique ($\phi = 26.5°$) fold axis distribution.

In general, the strain estimates obtained using the method described here are somewhat larger than those obtained using Sanderson's original approach. The most likely reason for the discrepancy lies with the method of comparing the observed and expected distributions. Sanderson uses coincidence of modes but this does not reflect variations in other regions, especially where frequencies are low. In contrast, the theoretical derivation of Pearson's classification and β_1 and β_2 contains implicitly a very rigorous comparison test which considers equally the whole range of the different distributions.

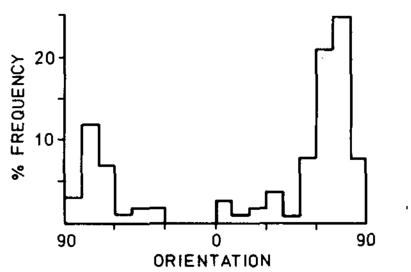

Fig. 4. Orientation frequency distribution histogram of angles between stretching lineation (X) and fold axes, Boscastle, Cornwall (modified from Sanderson 1973).

Strain-modified uniform distributions

Sanderson (1977) considered also the effect of homogeneous irrotational strain on orientation data with an initial uniform distribution. He showed that the uniform frequency distribution in a sector subtended by an angle α,

$$F = N\alpha/2\pi \tag{11a}$$

is modified to

$$F' = \frac{N}{2\pi}[\tan^{-1}(R_s \tan\theta_2') - \tan^{-1}(R_s \tan\theta_1')] \tag{11b}$$

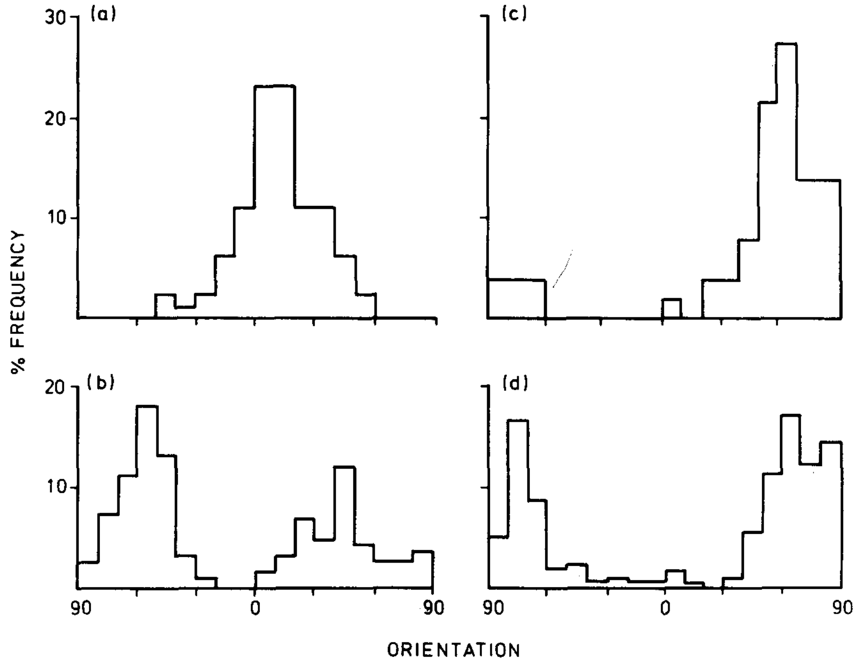

Fig. 5. Orientation frequency distribution histograms of angles between stretching direction (X) and fold axes from four localities in the Scottish SW Highlands (modified from Roberts & Sanderson 1974).

in an arc subtended by θ_1' and θ_2' measured from the maximum principal strain axis (X). In these equations, N is the sample size and R_s is the strain ratio ($R_s = (\lambda_1/\lambda_2)^{1/2}$). Initially the frequency is independent of orientation but as the strain increases a preferred orientation develops which is symmetrical about the extension direction (X) of the finite strain ellipsoid ($X \geqslant Y \geqslant Z$); the dispersion about X also decreases with increasing strain (Fig. 6).

Since the strain-modified uniform distributions are all symmetrical they plot along the β_2 (symmetry) axis of the β_1 vs β_2 graph, migrating away from the position of the theoretical uniform distribution ($\beta_1 = 0$, $\beta_2 = 1.8$) towards larger values of β_2 with increasing strain (Fig. 7). The displacement along the β_2 axis for each 0.1 increment of strain is considerable but approximately constant, even up to large values of strain. Thus, the graphs may be used to give sensitive estimates of strain over a wide range of values.

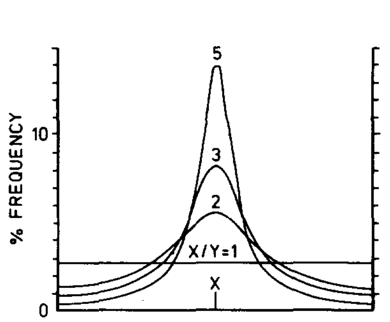

Fig. 6. Homogeneous irrotational strain-modified uniform orientation distributions (modified from Sanderson 1977).

Example of strain-modified uniform distributions

Beach (1980) studied the orientation of belemnites from Britain and France. He argued that in the undeformed state the orientation distribution of belemnites is approximately uniform (Fig. 8a) but, where the rock suffered an homogeneous irrotational strain, the belemnites show a preferred orientation symmetrical about the principal extension direction (Fig. 8c). Beach did not attempt any strain estimates from his examples but by using the technique described here all the distributions are shown to be symmetrical (Fig. 9) and hence probably due to homogeneous irrotational deformation. The undeformed example plots exactly at the position of the uniform distribution while an example thought by Beach to be only very slightly deformed (Fig. 8b) is found to have a strain ratio of 1.1. The example, said to be typical of the belemnite orientation distributions found in the French Maritime Alps (Fig. 8c), gives a strain ratio of 3.07.

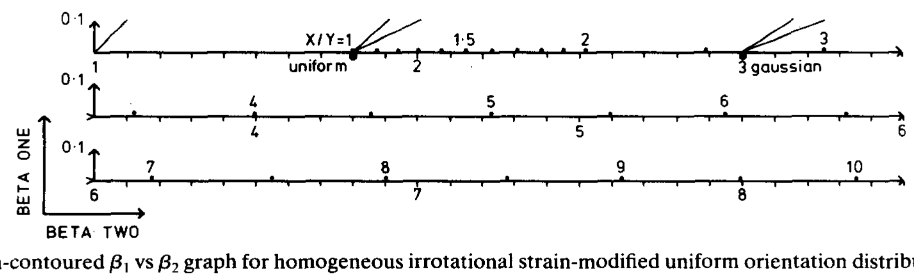

Fig. 7. Strain-contoured β_1 vs β_2 graph for homogeneous irrotational strain-modified uniform orientation distributions.

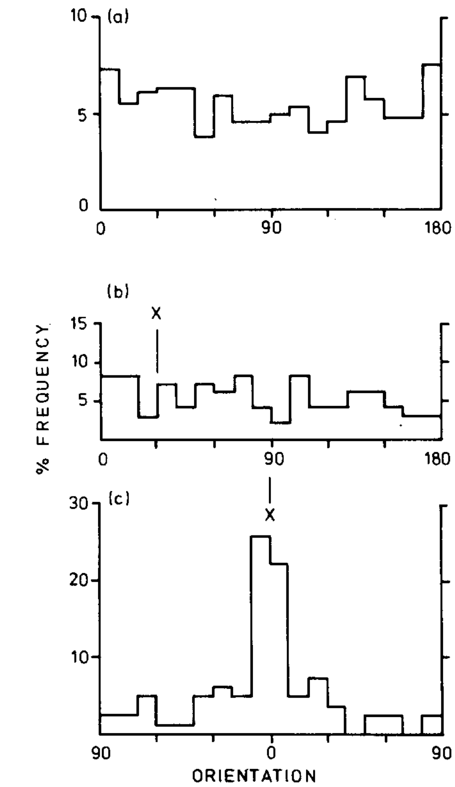

Fig. 8. Belemnite orientation frequency distribution histograms (after Beach 1980); (a) undeformed; (b) very slightly deformed; (c) deformed. See text for discussion.

DISCUSSION

(a) *Generality of the technique*

The technique described here is not itself a method of strain analysis. However, it does represent a very sensitive way of evaluating data from analytical methods which involve the modification of continuous frequency distributions by deformation. Although the examples considered concern only orientation data, homogeneous irrotational strain and Gaussian and uniform initial distributions, the technique is nevertheless applicable to any type of continuous data distribution and mode of deformation. The only requirement is that a model exists for the effects of a particular type of strain on a particular initial distribution.

In this respect it is worth noting that Sanderson (1977) did not really restrict his analysis of the strain modification of initially uniform distributions to irrotational strain, but considered the rotational components to play no part in determining the shape of the strain modified distribution, at least for passive markers. This was subsequently justified by Sanderson & Meneilly (1981, pp. 109–111) using Owens' (1973) development of the theory of strained angular density distributions (March 1932). In general, the form of the strain-modified frequency distribution is determined by the initial distribution (shape and orientation) and the deformation gradient tensor (relative to some defined reference frame). However, for the particular case of the initially uniform distribution, since its frequency is constant, the deformation gradient tensor can be factorized into a stretch tensor and a rotation tensor. The operation of the rotation tensor on a uniform distribution leaves the frequency unchanged and so irrotational and rotational deformations should produce similar strain-modified distributions (e.g. pure shear and simple shear result in the same strain-modified frequency distributions, D. J. Sanderson, personal communication 1982). Thus, distributions of belemnite orientations which are asymmetric with respect to the principal extension direction (Beach 1980, see Fig. 10 for examples), and therefore plot off the β_2 axis (Fig. 9), are more likely to be due to non-uniform initial distributions rather than rotational strains. It is possible that original sedimentary influences (e.g. palaeoslopes and/or current activity) were responsible for inducing slight preferred orientations such that the initial distributions responded as (platykurtic) Gaussian distributions.

(b) *Comparison of different data distributions*

The β_1 vs β_2 graphs (e.g. Figs. 3 and 7) show that if the type and magnitude of strain is constant then the shape of the strain-modified distribution is a function only of the initial distribution. Thus, if the fold axes and belemnites considered previously had occurred in the same region and had suffered similar amounts of homogeneous irrotational strain they would nevertheless show different frequency distributions. Furthermore,

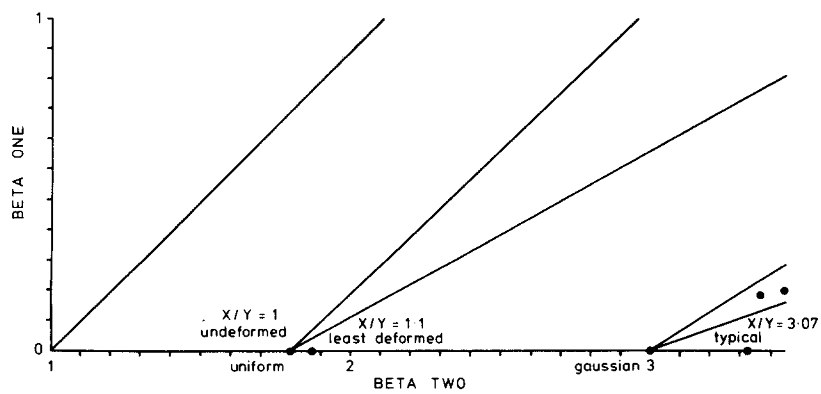

Fig. 9. Analysis of belemnite orientation frequency distributions using strain contoured β_1 vs β_2 graph for homogeneous irrotational strain-modified uniform orientation distribution. Examples plotted on the β_2 axis are symmetrical and therefore due to homogeneous irrotational strain of initially uniform orientation distributions. The other examples are probably due to deformation of initially nonuniform distributions.

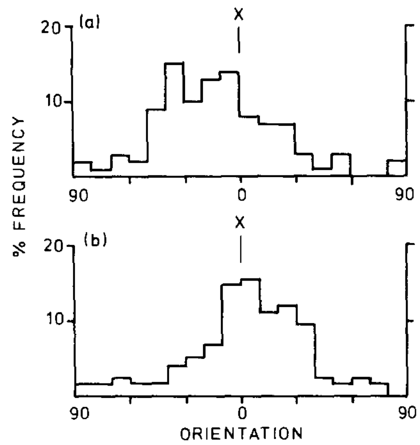

Fig. 10. Asymmetric belemnite orientation frequency distribution histograms (after Beach 1980).

although slight variations in homogeneous irrotational strain magnitudes have little effect on the modified shape of initially uniform distributions (see Fig. 7), initially Gaussian distributions can vary markedly. For example, using Fig. 3, a strain variation of $1 < X/Y < 3$ acting on an initial Gaussian distribution with $\mu^{1/2} = 20°$ and $\phi = 10°$ can produce distributions ranging from unimodal bell-shapes, through J or reverse-J shapes to bimodal shapes. In spite of this, it is still possible to determine accurate estimates of deformational parameters using the techniques described here.

(c) *Data collection*

While the use of β_1 and β_2 to classify continuous frequency distributions is theoretically soundly based, in practice it does involve several assumptions concerning the collection of the actual data. In particular, it is assumed that the data are a representative sample of the total population whereas in reality it is more likely to be either random or biased. The data distribution is therefore only an estimate of the total population distribution. Thus, the values of β_1 and β_2 calculated from the data are only estimates of the real values and hence do not necessarily define the best-fit theoretical distribution, which in turn means that the derived deformational parameters (i.e. X/Y and ϕ) may not be the true values. This problem is not a peculiarity of the β_1, β_2 technique but of the data collection process. It is therefore common to all methods of comparison and as such emphasises the *general* need for careful data collection. Since the β_1, β_2 technique involves more sample statistics than other goodness-of-fit tests it is to be preferred as a method of comparing observed and expected distributions.

It is also assumed that the pre-deformation distribution of the data is known. In particular, the initial standard deviation must be accurately defined since this influences the shape of the strain-modified distribution and determines the precise form of the β_1 vs β_2 graph to be used. If the standard deviation is not accurately defined then a range of strain-modified distributions is possible which consequently require different β_1 vs β_2 graphs for analysis. Thus, rather than a single estimate of the deformational parameters a range of values is obtained which may vary considerably depending on the imprecision in defining the initial standard deviation. Unfortunately, the definition of the pre-deformation data distribution may not be easy and in any case will be subject to similar data sampling problems as those discussed above.

Finally, although the examples discussed in this contribution involve orientation data, they have, nevertheless, been analysed using linear rather than circular sample statistics and distributions. It is therefore necessary to construct the histograms of the data about some external reference direction defined as zero orientation. The range of the histograms can then be considered to be $\pm 90°$ provided data $< -90°$ or $> +90°$ are transferred to the opposite regions of the histogram (e.g. 95° is plotted as $-85°$). This approach is convenient but ultimately should be replaced by a more rigorous technique based on the β_1, β_2 analysis of circular distributions.

CONCLUSIONS

(1) A method of strain analysis is described based on the modification of continuous frequency distributions by progressive deformation.

(2) Such distributions can be accurately described in terms of their shape via the dimensionless coefficients of skewness (β_1) and kurtosis (β_2); graphs of β_1 vs β_2 can therefore be used to distinguish different distributions.

(3) Theoretical studies of the effects of deformation on initial frequency distributions (i.e. strain-induced shape modifications) can be used to contour β_1 vs β_2 graphs in terms of strain.

(4) The positions of natural data distributions on these graphs consequently give the natural strain magnitudes.

Acknowledgements—The technique described here forms part of a statistical model of rock deformation currently under development. I should like to express my gratitude to Colin Ferguson for his interest, advice and encouragement during the course of this work. I should also like to thank Dave Sanderson for his comments on the application of this technique to his data. Joyce Heathcote typed the manuscript and Josie Wilkinson drew the figures.

REFERENCES

Beach, A. 1980. The analysis of deformed belemnites. *J. Struct. Geol.* **1**, 127–135.

Elderton, W. P. & Johnson, N. L. 1969. *Systems of Frequency Curves*. Cambridge University Press, London.

Harr, M. E. 1977. *Mechanics of Particulate Media: A Probabilistic Approach*. McGraw-Hill, New York.

March, A. 1932. Methematische Theorie der Regelung nach der Korngestalt bei affiner Deformation. *Z. Kristallogr.* **81**, 285–297.

Owens, W. H. 1973. Strain modifications of angular density distributions. *Tectonophysics* **16**, 249–261.

Roberts, J. L. & Sanderson, D. J. 1974. Oblique fold axes in the Dalradian rocks of the Southwest Highlands. *Scott. J. Geol.* **9**, 281–296.

Sanderson, D. J. 1973. The development of fold axes oblique to the regional trend. *Tectonophysics* **16**, 55–70.

Sanderson, D. J. 1977. The analysis of finite strain using lines with an initial random orientation. *Tectonophysics* **43**, 199–211.

Sanderson, D. J. & Meneilly, A. W. 1981. Analysis of three-dimensional strain-modified uniform distributions: andalusite fabrics from a granite aureole. *J. Struct. Geol.* **3**, 109–116.